1. BACKGROUND

At the 25 March 2010 HF Ops meeting, a talk was presented regarding Elevated Verticals. Two of the presentation slides showed a phasing diagram illustrating a direct wave and a reflected ground wave from an antenna, combining in the far field\(^1\), as either a phase additive Lobe or a phase canceling Null. The revised presentation is now posted on the NSARC website.

A discussion ensued as to the phase shifts that may or may not take place at the point of ground reflection. It was soon realized that no clear explanation was at hand to explain the reflection phenomena.

2. SUMMARY

Both polarizations have to be considered, Horizontal and Vertical E field polarization, as they behave differently.

Polarization is defined by the orientation of the Electric field with respect to the horizontal (the tangent to Earth’s surface). A horizontal dipole will radiate a horizontally polarized wave and a vertical antenna will radiate a vertically polarized wave.

Reflections can be summarized approximately as follows, ignoring grazing angle reflections, earth conductivity, and frequency for the moment,

- A Horizontally polarized wave, when reflected off ground, ALWAYS undergoes a phase shift, that is, the reflected wave is nominally 180 degrees out of phase with the incident wave.
- A Vertically polarized wave, when reflected off ground at angles greater than about 30 degrees, does NOT undergo a phase shift, that is, the reflected wave is in phase with the incident wave.

3. TECHNICAL DETAIL

3.1 Phasing

To see how antenna patterns form, an understanding of phasing is required. Phase describes how two waves relate to each other in terms whether their amplitudes add together or subtract from each other due to their phase differences. Figure 1 shows two waves, (a) where they are exactly in phase (0 degrees difference) and add, or (b) exactly out of phase (180 degrees difference) where they subtract and cancel.

\[\text{Exactly IN PHASE} \quad \text{EXACTLY Out of PHASE}\]

\[
\text{Amplitudes Add} \quad \text{Amplitudes Subtract}
\]

(a)

(b)

Figure 1 Phasing

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\(^1\) Don White & Associates. "Volume 3 Electromagnetic Interference and Compatibility". Chapter 5, pages 5.68 ff
Waves can combine at any in-between phase where they will be more or less additive or subtractive.

### 3.2 Antenna Radiation

An antenna, at some distant above earth ground, radiates energy at various vertical angles.

Energy radiated upwards away from ground is considered to be a Direct Wave. Energy radiated downwards towards earth and reflected off earth is considered to be a Reflected Wave.

![Figure 2](https://www.arrl.org/antenna-handbook)

**Figure 2**  
Path Length Differences

In Figure 2, the direct wave and the reflected wave are phase shifted from each other as a consequence of the two paths being of different lengths to any point in the far field, and also by phase shifts that may occur at the ground reflection point.

The two waves will combine with each other in the far field from 0 to 90 degree elevation, some elevations producing increases in signal strength, a Lobe, when they are phase additive, or combining at some other elevation exactly out of phase and canceling each other, a Null, producing little radiation.

The specific angles at which the lobes and nulls appear in the far field is primarily a function of the antenna radiation characteristics, its height above ground, and ground conductivity.

It turns out that vertically polarized and horizontally polarized reflections behave differently, and so the vertical elevation plots of horizontal antennas will be different than the vertical elevation plots of vertical antennas.

To predict the far field patterns, one has to know what the reflection phase shifts are for both reflected horizontal and vertical polarizations.

### 3.4 The Electromagnetic Wave

An electromagnetic wave consists of an electric field $E$ and a magnetic field $H$ that propagate in a certain direction $S$. "$S$" is known as the Poynting vector (much the same meaning and pronunciation as "pointing").

$E$ and $H$, constituting the electromagnetic wave, are in the same plane, but at right angles to each other. $S$ is at right angles to both of these and represents the direction the electromagnetic wave is moving. This is visualized in Figure 3.

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2 Corson and Lorrain “, Introduction to Electromagnetic Fields and Waves” Chapter 11, pages 387 ff
REFLECTIONS

The E and H field are represented as vectors. Vectors are drawn as “arrows”. The length of the vector indicates amplitude, in this case field strength, and the way in which it is pointing represents its direction. S points in the direction in which the wave propagating. This would be at the speed of light in free space.

![Figure 3: Electromagnetic Wave Field Vectors (not to scale)](image)

3.5 Reflection Rules

The physics of reflection follow some very specific rules. They are listed as follows,

Rule 1. The angle of reflection = angle of incidence (Snell’s Law)

![Figure 4: Reflection Law](image)

Rule 2. The voltage at the reflecting surface must equal zero

![Figure 5: Cancellation of Current and Voltage at Reflection](image)

Consider the reflecting surface to be a perfect conductor with R = 0.
The electric field of the incident wave induces a current at the reflecting surface, $I_{\text{incident}}$. The reflected wave E field must also have a current associated with it, $I_{\text{reflected}}$.

With no resistance in the reflecting surface, the current flowing would be infinite which is not possible, and so the reflected E field current must be equal and opposite to the incident E field current such that the currents cancel and $I_{\text{total}} = 0$, and E field at the surface = 0 as a consequence. This accounts for the phase reversal of the reflected wave in Figure 5.

Rule 3. When a reflection occurs, the Poynting vector will change direction, "... either the E or H vector must change direction in order that the Poyning vector can change direction ..". This requires either the E or H field to change phase by 180 degrees.

A certain protocol is required to explain the following diagrams since E, H and S are all at right angles to each other, i.e. 3 dimensional. Since we only have a 2 dimensional page on which to illustrate, the following notation is used to represent the 3rd dimension to the diagrams.

- ⦾ means that a vector points OUT of the Page
- ⦿ means that a vector points IN TO the page

### 3.6 Horizontally Polarized Reflection

The incident wave is angling down onto the reflecting surface. The electric field is horizontal to the reflecting surface. Technically, the E field is perpendicular to the plane of incidence. The S vector shows the direction of travel. The H field is pointing upwards and the E field is pointing IN to the paper away from the reader. At the point of reflection the two E field currents, which are parallel to the reflector as shown in Figure 5, are canceling each other out. The reflected E field thus changes phase by 180 degrees and the voltage at the reflecting surface = 0. Note the that reflected wave has the E field now pointed OUT of the paper satisfying the need to have either the H or E field reverse since the Poynting vector has changed direction. The reflected wave heads off as indicated by S which is at the same angle as the incoming wave was incident.

![Horizontally Polarized Reflection Diagram](image)

This illustrates the E field 180 degree phase shift that occurs on reflection.

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3 Corson and Lorrain “, Introduction to Electromagnetic Fields and Waves” Chapter 11, pages 391
3.7 Vertically Polarized Reflection

The incident wave is angling down onto the reflecting surface. The S vector shows the direction of travel. Notice that while the E field is slanted; it is not parallel to the reflecting surface. The E field is parallel to the plane of incidence, not the reflecting surface. To understand more easily how the reflection takes place, the slanted E field is resolved into two vectors, one being a vertical component Ev and the other being a horizontal component Eh tangential to the reflecting surface. The H field is pointing OUT of the paper towards the reader.

At the point of reflection the two tangential field currents due to Eh incident, and Eh reflected, as explained in Section 3.6, cancel each other out. The reflected wave heads off as indicated by S at the same angle as the incoming wave was incident. The reflected E field has not undergone a phase change and is still in the same plane as the incident E field. However, the H field is now pointing IN to the paper to satisfy the need to have a reversal of one of the two fields.

![Figure 6: Vertically Polarized Reflection](image)

This shows that the E field does not reverse and that the H field undergoes a 180 degree phase shift on reflection.

3.8 Ground Loss

The Earth’s surface is not a perfect conductor. Reflections occur off soils, rock and water surfaces. As a result, the reflection is not complete and incident wave energy flows into the surface material and is lost. The reflected wave thus has less energy than the incident wave and so the reflected wave is attenuated to some extent. As the earth becomes more lossy, depending on the conductivity and dielectric constant, the reflected wave will become more attenuated thus affecting the far field Nulls and Lobes.

The Reflection Coefficient “ρ” is a measure of the ratio of the amplitude of the reflected wave to the amplitude of the incident wave and indicates the reduction in amplitude of the reflected wave. A perfect conductor would have ρ = 1 which indicates all incident energy is reflected whereas ρ = 0 indicates all incident energy is transmitted into the reflecting material.

3.9 Vertically Polarized Reflection from Real Ground

For vertically polarized waves, ground loss affects both amplitude and phase particularly when the incident angle becomes shallow. The amount of phase shift and amplitude reduction depends on frequency, ground conductivity and the angle of incidence of the wave.

---

For a highly conductive surface, such as salt water, reflection is much higher the losses are minimal compared to rock or a desert.  

![Figure 7](image_url)

**Figure 7**
Effect of Ground on Phase and Amplitude

At a particular angle of incidence, however, a vertically polarized wave is not reflected and most energy is transmitted into the ground surface. This angle of incidence is the Brewster angle, $\theta_B$.  

In Figure 7 is plotted over earth with a typical conductivity of 0.005 S/m; and a dielectric constant of 13. The Brewster angle for this surface is about 15 degrees where the reflection coefficient $\rho$ goes to near zero. Over seawater the Brewster angle decreases significantly from 15 degrees to about 2 degrees due to the much higher conductivity = 5 S/m and a dielectric constant of 81. With such a low Brewster angle a vertically polarized antenna will have a lower angle lobe when traveling over salt water due to higher conductivity.

Figure 8 is the same representation as Figure 7 but showing the phase change at low incident angles and the effect of frequency. The vertical reflection of the E field starts to change noticeably when the incident angle starts to fall below 30 degrees. At about 15 degrees the phase has changed 90 degrees and at zero degrees it is at 180 degrees.

![Figure 8](image_url)

**Figure 8**
Vertical Polarization Phase Angle Change with Incident Angle and Frequency

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5 Davies. “Ionospheric Radio” Page 452- 453
6 ARRL Handbook. 19th edition. Table 2, page 3-6
7 http://en.wikipedia.org/wiki/Brewsters_angle
8 ARRL Handbook. 19the edition. Table 2, page 3-6
9 Robert Collin “Antennas and Radio Wave Propagation”. Pages 341 ff
3.10 Impact of Low Angle Phenomena for Vertical Antennas

If the earth were perfectly conducting, there would be no phase shift at the surface of the earth. The direct wave and any slightly downward radiated wave would be much in phase and add together producing a maximum lobe. However, real earth causes phase shifts as noted and this appears in vertical radiation plots as a Null developing typically at 15 degrees and increasing at lower angles over average ground. If the antenna were over seawater, the Null would start at < 5 degrees. In both cases the Null is complete at 0 degrees.

3.11 Horizontally Polarized Reflections from Real Ground

Horizontally polarized waves that approach low incident angles are subject to the phase and amplitude changes as well but very much less so as seen in Figure 9.

![Figure 9](image)

Effect of Ground on Phase and Amplitude

Figure 10 is the same representation of the phase change at low incident angles but showing the effect of frequency. Again, phase shift is relatively minimal.

![Figure 10](image)

Horizontal Polarization Phase Angle Change with Incident Angle and Frequency

It is interesting to note that the phase shift is 180 degrees only for a perfectly grazing angle of 0 degrees. As the angle increases, the phase change approaches 190 degrees at a full perpendicular angle of incidence.

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10 ARRL Antenna Handbook, 19th Edition
11 Robert Collin “Antennas and Radio Wave Propagation”. Pages 341 ff
3.12 Acknowledgements

Thanks to the club members who pitched in with comments, discussions and references, particularly Bill Tracey VE7QQ and Nick Massey VA7NRM, to resolve and make clear what actually happens at the boundary.

4 ANTENNA FAR FIELD

Appendix III illustrates the formation of Nulls and Lobes of radio waves due to the direct wave and the ground reflected wave adding in the far field. The phasing of these waves produces the far field elevation plots.

Both horizontal and vertical polarization are taken into account to show their different reflection behavior. The horizontally polarized waves are shown to undergo the 180 degree phase shift while the vertically polarized wave is not phase shifted.

Note that the elevation plots for a dipole (horizontal polarization) and NOT the same as for a vertical antenna (vertical polarization) system ELEVATED above ground and at the same height as the dipole.

Certain liberties are taken in these simplified diagrams. The vertically polarized illustrated on page 4 does not take into account the beginning of the low angle phase shift that may be expected. The surface is considered to have a reflection coefficient of 1, and the effect of near field effects is neglected. Appendix IV provides some information on this subject. While Near Field – Far Field wave impedance is not a factor in all this, Appendix V provides insight in this matter as well just for interest.

Appendices

Appendix I Electromagnetics. Excerpts from Corson and Lorrain, “Introduction to Electromagnetic Fields and Waves”

Appendix II Excerpts from Robert Collin. “Antennas and Radiowave Propagation”.

Appendix III Antenna Reflections


These references may be out of print.
Three cases of reflection of an incident wave from a conductive surface are presented.

1. **Section 11.5.2 Horizontal Electric Field Polarization - Figure 11-22**
   The electric field is perpendicular (normal) to the plane of incidence = horizontal polarization.
   
   The electric field is phase shifted **180 degrees**

2. **Section 11.5.3 Vertical Electric Field Polarization - Figure 11-23**
   The electric field is parallel to the plane of incidence = vertical polarization.
   
   The electric field is **NOT phase shifted**

3. **Section 11.5.4 Normal Incidence - Figure 11-24**
   The electric field is perpendicular (normal) to the reflecting surface.
   
   The electric field is **phase shifted 180 degrees**
\[ n_i (E_{ai} + E_{ai}) \sin \theta_i = K_{ni} H_{aai} \]  
and  
\[ k_{2a} H_{aai} + k_{2a} H_{aai} = 0. \]

From the last equation,  
\[ H_{aai} \gg H_{aai} \]  
since \( k_{2a} \) is much smaller \( k_{2a} \) (Eq. 11-132). The wave in the conductor therefore has its \( \mathbf{E} \) vector parallel to the \( y \)-axis and its \( \mathbf{H} \) vector parallel to the \( x \)-axis. It propagates in the negative direction of the \( z \)-axis.

From the above,  
\[ \left( \frac{E_{ex}}{E_{ei}} \right) = \frac{n_1 K_{exc} \cos \theta_i - \frac{\lambda_0}{\delta} (1 - f)}{n_1 K_{exc} \cos \theta_i + \frac{\lambda_0}{\delta} (1 - f)} \]  
(11-139)

We would again have obtained the same result by the direct application of Fresnel’s equation (11-21) with \( \cos \theta_i \) as in Eq. 11-127.

We have shown above in Eq. 11-131 that \( \lambda_0 / \delta \). Then  
\[ \frac{E_{ex}}{E_{ei}} \approx -1, \]  
(11-140)

at least if \( K_{ni} \) is much smaller than \( \lambda_0 / \delta \). The coefficient of reflection \( R \) is thus approximately equal to unity, and \( \mathbf{E} \) is reflected \( \pi \) radians out of phase as in

Figure 11-22

The incident, reflected, and transmitted waves at the interface between a dielectric and a good conductor. The incident wave is in the dielectric and is polarized with its \( \mathbf{E} \) vector normal to the plane of incidence. The \( \mathbf{H} \) vector of the transmitted wave is parallel to the interface, as shown, but it lags the \( \mathbf{E} \) vector by \( \pi/4 \) radians, from Eq. 10-105. The coefficient of reflection \( R \) is approximately equal to unity; the electric fields of the incident and reflected waves nearly cancel on the surface and a weak, highly attenuated wave penetrates perpendicularly into the conductor.

Figure 11-22. This is not surprising, since the electric field in the conductor must be expected to be quite small.

It is interesting to note that there is some loss of intensity on reflection from a good conductor, since \( E_{ai} \) is somewhat smaller than \( E_{ai} \). In total reflection,
Reflection and Refraction at the Surface of a Good Conductor

On the other hand, there is no loss of intensity and the coefficient of reflection is exactly equal to unity from Eq. 11-104.

Also, from Eq. 11-133,

$$\left( \frac{E_{\text{tot}}}{E_{\text{si}}} \right)_n = \frac{2n_1K_{\text{ext}} \cos \theta_i}{n_1K_{\text{ext}} \cos \theta_i + \frac{\lambda_0}{\delta} (1 - j)} \ll 1. \quad (11-141)$$

The transmitted wave is therefore both relatively weak and, as we have seen, highly attenuated. Solving for $H_{\text{ext}}$ shows that $E$ and $H$ are related as in Eq. 10-105, which applies to plane waves in good conductors.

Reflection from the surface of a dielectric with $n_2 \gg 1$ would also lead to Eq. 11-140 (see Eq. 11-21) and to a weak transmitted wave.

11.5.3. The Electric and Magnetic Fields in the Reflected and Transmitted Waves for the Case of Reflection at the Surface of a Good Conductor. Incident Wave Polarized with Its $E$ Vector Parallel to the Plane of Incidence.

It is found similarly that

$$\left[ \frac{E_{\text{tot}}}{E_{\text{si}}} \right]_p \approx -1. \quad (11-142)$$

The three waves are as shown in Figure 11-23. The negative sign in the above equation means that the $E$ vector of the reflected wave is in the direction oppo-

![Figure 11-23](image)

Reflection and refraction at the interface between a dielectric and a good conductor. The incident wave is in the dielectric and is polarized with its $E$ vector parallel to the plane of incidence. The $E$ vector of the transmitted wave is as shown, but it leads the $H$ vector by $\pi/4$ radians. Just as in Figure 11-22, the coefficient of reflection $R$ is approximately unity; the tangential components of the $E$ vectors of the incident and reflected waves nearly cancel on the interface; and a weak attenuated wave penetrates perpendicularly into the conductor.

The transmitted wave is again a weak, highly attenuated plane wave which penetrates perpendicularly into the conductor.
One interesting application of the above discussion is the problem of communication with submarines at sea. For shore-to-ship communication with the submarine antenna submerged, the efficiency is extremely low, first because most of the incident energy is reflected upwards at the surface of the sea, and second because the weak transmitted wave is highly attenuated. It was shown in Problem 10-5 that the attenuation is about 17 decibels/foot at a frequency of 20 megacycles per second and 1.7 decibels/foot at 20 kilocycles per second. Very low frequencies (VLF) are used at high power. The response of a submerged receiving antenna will be calculated in Problem 13-20. Communication in the direction ship-to-shore is presently impossible at these low frequencies, since the power required at the transmitter is too large, and since it is not possible to use a sufficiently large antenna on the submarine. Two-way communication is achieved at frequencies of a few megacycles per second with the submarine antenna projecting from the water.

11.5.4. Reflection and Refraction at the Surface of a Good Conductor at Normal Incidence. At normal incidence, $\theta_i = 0$, and, from Eq. 11-139,

$$\frac{E_{re}}{E_{im}} = \frac{K_{\omega_0} - \frac{\lambda_0}{n_0 \delta} (1 - j)}{K_{\omega_0} + \frac{\lambda_0}{n_0 \delta} (1 - j)}.$$  \hspace{1cm} (11-143)

We again have $E_{re} \approx -E_{im}$ for a good conductor, that is, for $\lambda_0 \gg \delta$.

The electric field intensity $E$ of the reflected wave is opposite that of the incident wave, or approximately so. This makes for a weak electric field intensity in the conductor, since the tangential component of $E$ is the same on either side of the interface.

![Diagram](image)

**Figure 11-24**

*Reflection at normal incidence from the surface of a good conductor. The electric fields of the incident and reflected waves cancel at the interface, or nearly so. The magnetic field intensities add, however, with the result that the amplitude of the $H_{im}$ vector at the interface is $2H_{im}$."

The incident and reflected waves are shown in Figure 11-24. Since the direction of the reflected wave is opposite to that of the incident wave, and since the vector product $E \times H$ must always give the direction of propagation, the
[11.5] Reflection and Refraction at the Surface of a Good Conductor

magnetic field intensity of the reflected wave must be in phase with that of the incident wave, as shown in the figure.

It is interesting to consider the standing wave pattern which is produced by the reflection of an electromagnetic wave from a good conductor at normal incidence. At the reflecting surface the electric field intensities nearly cancel, and we have a node of \( E \); the magnetic field intensities add, and we have a loop of \( H \). This is shown in Figure 11-25. The nodes of \( E \) and of \( H \) are thus not coincident but are spaced a quarter wavelength apart.

**Figure 11-25.** The standing-wave pattern resulting from the reflection of an electromagnetic wave at the surface of a good conductor. The curves show the standing waves of \( E \) and of \( H \) at some particular time. The nodes of \( E \) and of \( H \) are not coincident but are spaced \( \lambda/4 \) apart as shown. It is shown in Problem 11-18 that \( H \) leads \( E \) by \( \pi/2 \), thus the maximum values of \( E \) and of \( H \) do not occur at the same time.

A little thought will show that a similar situation exists for reflection from any surface. Either the \( E \) or the \( H \) vector must change direction on reflection, in order that the Poynting vector \( E \times H \) can change direction.
ANTENNAS
AND RADIOWAVE
PROPAGATION

Robert E. Collin
Case Western Reserve University

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The general features of the interference phenomena associated with antennas placed over the earth can be determined by studying the effects associated with antennas located above a flat earth. Figure 6.1 shows a transmitting antenna at height $h_1$ and a receiving antenna at height $h_2$, with separation $d$. The figure also shows the direct ray and indirect or reflected ray that reach the receiving antenna. When the two path lengths $R_1$ and $R_2$ differ by an appropriate amount there may be either constructive or destructive interference at the receiving antenna.

With reference to Fig. 6.1, the field that reaches the receiving antenna directly will produce a voltage proportional to

$$f_1(\theta_1)f_2(\theta'_1)\frac{e^{-jk_0R_1}}{4\pi R_1}$$

where $f_1$ and $f_2$ are the radiation field strength patterns of the two antennas. The voltage produced by the indirect wave is proportional to

$$f_1(\theta_1)f_2(\theta'_2)\rho e^{j\phi}\frac{e^{-jk_0R_2}}{4\pi R_2}$$

where $\rho e^{j\phi}$ is the reflection coefficient at the ground. In the usual situation $h_1$ and $h_2$ are very small compared with the separation $d$, so the angles $\theta_1$, $\theta_2$, $\theta'_1$, $\theta'_2$ are very small, and the antenna radiation patterns can be assumed constant over the range of angles involved. An exception would be the case when highly

![Figure 6.1 Illustration of direct and reflected rays.](image)
directive antennas are used and \( h_a \) is large, such as occurs if the transmitting antenna is located on the ground and the receiving antenna is located aboard a high-flying aircraft. In this case very little power might be radiated toward the ground; that is, \( f_1(\theta_3) \ll f_1(\theta_1) \). The total received voltage will be proportional to (we use \( R_2 = R_1 \) in the denominator)

\[
\left| f_1(\theta_3)/f_2(\theta_1) \right| \frac{e^{-j\phi R_1}}{4\pi R_1} \left[ 1 + \rho e^{j\phi} \frac{f_1(\theta_2)f_2(\theta_3^*)}{f_1(\theta_1)f_2(\theta_1^*)} e^{-j\phi(R_2-R_1)} \right] = \left| f_1(\theta_3)/f_2(\theta_1) \right| \frac{e^{-j\phi R_1}}{4\pi R_1} |F|
\]

(6.1)

The factor \( F \), called the path-gain factor, shows how the field at the receiving antenna differs from the value it would have under free-space propagation conditions. When it can be assumed that \( f_1(\theta_3) = f_1(\theta_1) \) and \( f_2(\theta_3^*) = f_2(\theta_1^*) \), then \( F \) can be expressed as

\[
F = |1 + \rho e^{j\phi} e^{-j\phi(R_2-R_1)}|
\]

(6.2)

The path-gain factor is the array factor associated with the antenna at height \( h_1 \) and its image below the surface, with the relative excitation of the image antenna being \( \rho e^{j\phi} \).

With reference to Fig. 6.1, it can be seen that \( R_1 = [d^2 + (h_a - h_1)^2]^{1/2} \) and \( R_2 = [d^2 + (h_a + h_1)^2]^{1/2} \). When \( h_1 \) and \( h_a \) are very small compared with \( d \), a binomial expansion gives

\[
R_1 \approx d + \frac{1}{2} \frac{(h_a - h_1)^2}{d} \quad R_2 \approx d + \frac{1}{2} \frac{(h_a + h_1)^2}{d}
\]

from which we obtain

\[
R_2 - R_1 = \frac{2h_1 h_a}{d}
\]

If \( \rho e^{j\phi} \) were equal to \(-1\) then

\[
F = \left| 1 - e^{-j\phi h_1 h_a d} \right| = 2 \left| \sin \frac{k_0 h_1 h_a d}{d} \right|
\]

(6.3)

This shows that interference effects can lead to a doubling of the field strength relative to its value under free-space conditions. With reference to Fig. 6.2 we let \( \phi_0 \) be the elevation angle given by \( \tan \phi_0 = h_a / d \) so that Eq. (6.3) can be written as

\[
F = 2 \left| \sin(k_0 h_1 \tan \phi_0) \right|
\]

(6.4)

The relationship expressed by Eq. (6.4) is usually plotted in the form of a coverage diagram showing the variation of \( F \) with \( h_a / d \), that is, with \( \phi_0 \), for given values of \( h_1 \) and \( k_0 \). Note that \( F \) is a maximum when

\[
\tan \phi_0 = \frac{1}{k_0 h_1} \left( \frac{\pi}{2} + n\pi \right) = \frac{\lambda_0}{h_1} \left( \frac{1}{2} + \frac{n}{2} \right) \quad n = 0, 1, 2, \ldots
\]

(6.5a)
and is a minimum when
\[ \tan \psi_0 = \frac{\lambda_0 n}{h_1/2} \quad n = 0, 1, 2, \ldots \]  
\hspace{1cm} (6.5b)

A coverage diagram is a plot of the relative field strength as a function of direction in space from the transmitting antenna. It is analogous to the field-strength radiation pattern of an antenna. In any coverage diagram the fixed parameters are the height \( h_1 \) of the transmitting antenna and the wavelength \( \lambda_0 \). The distance \( d \) to the location of the receiving antenna and the height \( h_2 \) of the receiving antenna are variable parameters, and each pair of values \( h_2, d \) determines a point in space. The coverage diagram is a plot of the curves \( F/r = \text{constant} \) in the \( h_2, d \) plane. In most situations the direct line-of-sight distance \( r \) between antennas is very nearly equal to the horizontal distance \( d \). The various curves of \( F/r \) that are plotted are usually chosen to represent the same signal level that would be obtained at a distance of a multiple or a fractional multiple of a convenient free-space reference range \( r_f \); for example, \( F/r = m/r_f \) or \( F = mr/r_f = md/r_f \), with \( m = 1, \sqrt{2}, 2, \ldots \) or \( 1/\sqrt{2}, 1/2, \ldots \). The difference in signal level between successive curves is then 3 dB. By using Eq. (6.3) we find that the constant signal level curves are given by (we assume that \( r = d \))
\[ F = 2| \sin k_0 h_1 h_2 | = m \frac{d}{d_f} \]  
\hspace{1cm} (6.6a)

when the reflection coefficient equals -1. For the flat-earth case it is more convenient to use Eq. (6.4) or (6.5) which gives
\[ 2| \sin k_0 h_1 \tan \psi_0 | = 2| \sin k_0 h_1 \psi_d | = m \frac{d}{d_f} \]  
\hspace{1cm} (6.6b)

In this equation \( d \) can be treated as the radial coordinate and \( \psi_0 \) as the angle coordinate in a polar-coordinate reference frame. However, note that since the vertical scale representing \( h_2 \) is usually expanded relative to that for \( d \), the angle \( \psi_0 \) appears much larger than it actually is.

Whenever \( h_1 \gg \lambda_0 \) and \( n \) is small, \( \tan \psi_0 \approx \psi_0 \) and the above relations show that the lobe structure is very fine; i.e., the angular separation between lobes is very small. For example, if \( h_1 = 100\lambda_0 \), then the lobes are separated by \( \lambda_0/2h_1 = 1/200 \) rad, or by approximately 0.3°. Figure 6.3 shows a typical coverage diagram. If \( r_f \) is the free-space range for a given received signal strength, then with interference taken into account the maximum range is \( 2r_f \), which corresponds to a horizontal distance \( d = 2r_f \cos \psi_0 \). For small values of \( \psi_0 \)
we have \( d = 2r_f \). The curves corresponding to \( d = 2r_f \cos \psi_0 \) appear as vertical lines in Fig. 6.3 because of the greatly expanded vertical scale. The coverage diagrams shown in Fig. 6.3 are plotted for a free-space propagation distance of 2 km. Any pair of values of \( h_2 \) and \( d \) that lies on the curve describing a lobe represents a point in space where the received signal strength is the same as it would be at a distance of 2 km under free-space propagation conditions. The smaller lobe shown in Fig. 6.3b represents a constant signal level 3 dB greater than that of the larger lobe and comes from using \( m = \sqrt{2} \) in Eq. (6.6b).

When the-coverage diagram has been plotted it is a simple matter to determine the field strength at the receiving antenna relative to the free-space value. For example, if the receiving antenna height is 10 m, Fig. 6.3b shows that the received signal strength at a distance of 3.2 km is the same as that at 2 km under free-space conditions. The same figure shows that by raising the antenna height to 25 m at a distance of 4 km a maximum signal level will be received. This signal level will be the same as that at 2 km with free-space propagation.

When the angle \( \psi_0 \) is considerably below the first lobe maximum, Eq. (6.4)
can be approximated by $2k_0h_1\psi_0$, so

$$F = 2k_0h_1\psi_0 = \frac{2k_0h_1h_2}{d}$$ (6.7)

which makes the received signal voltage vary as the inverse square of the distance, thus reducing the maximum useful range quite severely.

The coverage diagrams shown in Fig. 6.3 are based on taking $\rho = 1$, $\phi = \pi$. In practice this is a good approximation for the reflection coefficient for both horizontal and vertical polarization when the grazing angle $\psi$ is small, say, 1° or less. When $\psi$ is larger than 1°, $\rho e^{i\phi}$ may depart appreciably from −1 for vertical polarization but may still be approximated by −1 for horizontal polarization for values of $\psi$ up to 10° or more.

The reflection coefficient $\rho e^{i\phi}$ is given by the Fresnel expressions for the reflection coefficients for a plane TEM wave polarized with the electric field in the plane of incidence (vertical polarization) and for a wave polarized with the electric field perpendicular to the plane of incidence (horizontal polarization). The Fresnel reflection coefficients depend on the ground conductivity, permittivity, frequency, and angle of incidence. If the ground conductivity is $\sigma$, the permittivity $\epsilon$ is $\kappa \epsilon_0$, and $\psi$ is the grazing angle of incidence, then

$$\rho e^{i\phi} = \frac{(\kappa - j\chi) \sin \psi - \sqrt{(\kappa - j\chi) - \cos^2 \psi}}{(\kappa - j\chi) \sin \psi + \sqrt{(\kappa - j\chi) - \cos^2 \psi}} \quad \text{vertical polarization} \quad (6.8a)$$

$$\rho e^{i\phi} = \frac{\sin \psi - \sqrt{(\kappa - j\chi) - \cos^2 \psi}}{\sin \psi + \sqrt{(\kappa - j\chi) - \cos^2 \psi}} \quad \text{horizontal polarization} \quad (6.8b)$$

where $\chi = \sigma / \omega \epsilon_0$. Typical values for the dielectric constant $\kappa$ are around 15, while the conductivity $\sigma$ may range from $10^{-3}$ to $3 \times 10^{-2}$ S/m, with $10^{-2}$ S/m being a typical value for flat prairie land. The conductivity of mountainous regions is much lower. In general, $\kappa$ is smaller, around 6 or 7, for soil with poor conductivity and will increase up to about 30 for soil with a high conductivity.

Figure 6.4 shows the behavior of $\rho$ and $\phi$ as a function of the grazing angle $\psi$. Of particular significance is the Brewster angle effect for vertical polarization, which causes $\rho$ to go through a minimum for values of $\psi$ below about 15°. As $\rho$ moves through the minimum with decreasing values of $\psi$, the phase angle $\phi$ undergoes a rapid change from near 0° to 180°. This effect makes $\rho e^{i\phi}$ nearly equal to −1 for both vertical and horizontal polarizations when the grazing angle $\psi$ approaches zero. For a perfectly conducting surface $\rho e^{i\phi}$ would equal +1 for vertical polarization. As the frequency $\omega$ increases, the effect of a finite ground conductivity decreases, since the parameter $\chi = \sigma / \omega \epsilon_0$ decreases. Thus for frequencies above 50 MHz, the ground behaves very nearly like a dielectric medium, since the displacement current $j\omega \epsilon E$ is then much larger than the conduction current $\sigma E$. If the point of reflection occurs over water, particularly seawater, the reflection coefficient can be approximated by −1 for horizontal
polarization but may differ significantly from $-1$ for vertical polarization, as reference to Fig. 6.5 shows. In the case of a rough sea the reflection coefficient could be quite small for either polarization.

Whenever the point of reflection occurs over a rough surface the field is scattered in a more diffuse manner, and the specular reflected component, and hence $\rho$, is reduced in value. A measure of the height of the surface irregularities that constitute a “rough surface” may be obtained by considering the effective wavelength of the incident wave in the direction perpendicular to the surface. If $z$ is the coordinate perpendicular to the surface and $x$ is the coordinate along the surface, the incident wave will have a propagation factor $e^{i k_0 z \sin \psi}$ and $e^{-i k_0 z \sin \phi}$. Thus in the vertical direction the effective wavelength $\lambda_e$ is given by

$$\lambda_e = \frac{2\pi}{k_0 \sin \psi} = \frac{\lambda_s}{\sin \psi} \quad (6.9)$$
Figure 6.5 Reflection coefficient for vertical polarization for seawater. The marked angles are the Brewster angles when the conductivity is zero. (From D. E. Kerr, Propagation of Short Radio Waves, McGraw-Hill Book Company, New York, 1951.)
When the grazing angle of incidence is small, $\lambda_o$ will be large compared with $\lambda_o$, often by a factor of 10 to 100. If the point of reflection is raised by an amount $\lambda_o/10$ the change in phase of the reflected wave reaching the receiving antenna will be $(2k_o \sin \psi)\lambda_o/10 = 0.4\pi$. This may be regarded as being the boundary between what can be considered to be a rough surface and a smooth surface.†

With this criterion the surface of generally flat land can be considered “smooth” whenever the surface irregularities have an average height variation of $\lambda_o/10 \sin \psi$. For example, with $\lambda_o = 1 \text{ m}$ and $\psi = 1^\circ$, we find that height variations of up to $6 \text{ m}$ can still be regarded as a smooth surface. At the longer wavelengths most surfaces appear to be smooth, but at microwave frequencies most surfaces would be rough and the reflection coefficient would be smaller than that given by the Fresnel formulas.

A complication that has not been included in the flat-earth interference formulas is the effect of the decrease in the index of refraction of the atmosphere with height above the surface.‡ At greater heights the less dense atmosphere results in a smaller index of refraction. This has the effect of causing the ray that leaves the antenna at a finite angle relative to the ground to curve or bend in a downward direction in accordance with Snell’s law of refraction. The phenomenon of ray curvature may be readily understood by dividing the atmosphere into layers, with discrete values for the index of refraction in each layer, as shown in Fig. 6.6. For this staircase approximation to the continuous variation in the index of refraction, Snell’s law gives

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 = \cdots n_n \sin \theta_n = \cdots$$

Thus since each successive value of $n_i$ is smaller than the preceding value, the angles $\theta_n$ must increase and the ray curves in the downward direction. For propagation over a spherical earth this ray curvature extends the radio horizon beyond the geometrical horizon.

† The Rayleigh criterion allows for an obstruction with a height of $\lambda_o/8$ leading to a maximum phase change of $0.5\pi$.

‡ The decrease in the refractive index with height is not always monotonic. Inversion layers leading to a phenomenon known as ducting can occur. Such effects are discussed in Sec. 6.12.
The effect of ray curvature can be taken into account in a simple way for propagation over a spherical earth by replacing the earth with an earth having a larger radius and considering the rays to propagate along straight lines, provided the index of refraction decreases linearly with height. By means of this artifice the height of any point on the ray above the surface of the earth remains the same. For propagation studies a standard index-of-refraction profile is commonly chosen such that it is equivalent to increasing the radius of the earth by a factor of 4/3. Thus the effective earth’s radius \( a_r \) is chosen to be 5280 mi (8497 km). With reference to Fig. 6.7, it is seen that \((h_1 + a_r) = R^2 + a_r^2\), so \(R^2 = 2h_1a_r + h_1^2 = 2h_1a_r\). Since the antenna height \(h_1\) is small compared with the distance to the horizon, the slant distance \(R\) is nearly equal to the horizontal distance \(d_r\) to the horizon. Thus the distance to the horizon is given by \(d_r = (2h_1a_r)^{1/2}\), and if \(d_r\) is expressed in miles and \(h_1\) in feet we have

\[
d_r \text{ mi} = \sqrt{2h_1}\text{ ft}
\]

(6.10)

The maximum line-of-sight distance in miles between two antennas at heights \(h_1\) and \(h_2\) ft above a spherical earth with standard refraction conditions is then readily seen to be given by

\[
d_{\text{max}} = \sqrt{2h_1} + \sqrt{2h_2}\text{ mi}
\]

(6.11)

The flat-earth interference formulas are generally not valid for distances approaching the maximum horizontal line-of-sight range. The exact distance over which the flat-earth formulas can be used depends on a number of factors, including wavelength. It is difficult to establish the range of validity without direct comparison with the interference effects based on using a spherical earth model. The evaluation of interference effects over a spherical earth is considerably more complex than that for a flat earth and is discussed in the next section.
APPENDIX III
ANTENNA ELEVATION PLOTS

HORIZONTALLY POLARIZED E FIELD
NULL PATTERNS

Out of Phase Wave Front
Null at 90 degrees

180 degree Phase Shift of
Reflected E field

Out of Phase Wave Front
Lobe at 30 degrees

Cancellation
Null at 30 degrees

Ground

Lobe at 50 degrees
Null at 30 degrees

HORIZONTALITY POLARIZED E FIELD
LOBE PATTERN

Additive
Lobe at 50 degrees

In Phase Wave Front
Lobe at 50 degrees

180 degree Phase Shift of
Reflected E field

Ground

Lobe at 50 degrees
Null at 30 degrees
VERTICALLY POLARIZED E FIELD
NULL PATTERNS

NULL at 90 degrees is due to 0 radiation off end of vertical antenna

0 degree Phase Shift of Reflected E field

Elevated Vertical Elevation Plot
Null at 42 degrees
Lobe at 25 degrees

VERTICALLY POLARIZED E FIELD
LOBE PATTERN

Additive Lobe at 50 degrees

In Phase Wave Front Lobe at 25 degrees

Elevated Vertical Elevation Plot
Null at 42 degrees
Lobe at 25 degrees
APPENDIX IV

NEAR FIELD – FAR FIELD TRANSITION


A.1 Transition Distance

The distance for which far-field approximations are no longer valid and for which near-field considerations must be applied is called the transition distance. Basically the transition from near-field to far-field conditions is a gradual one. However, by specifying the error in the far-field pattern as one moves closer to the antenna, a specific transition distance relationship can be obtained.

One criterion used to define transition distance is to limit the phase error to 1/8 of a wavelength.* This corresponds to about a 1 dB error in gain obtainable at an arbitrarily large distance. Now consider the antenna configuration of Fig. 5.28. The distance to a field point along the normal axis of the antenna is different from the distance between the edge of the antenna and the field point. This difference is denoted the space-phase error.

Assuming that the antenna dimension, $b$, is large compared to the wavelength (i.e., $b \gg \lambda$), in order to limit this error to $\lambda/8$, the distance, $R$, to the field point from the antenna must satisfy:

$$R < \frac{\lambda^2}{\lambda}, \text{ for } \lambda >> \lambda$$

(5.31)

Eq. (5.31) applies for high and medium-gain antennas. When this requirement is not met (i.e., $b$ is not $\gg \lambda$, for low-gain antennas), the equation is no longer valid, and to assume far-field conditions it is necessary to adopt the criterion:

$$R > 3\lambda, \text{ for } \lambda \leq \lambda$$

(5.32)

* See Sec. 3.1, Vol. 4, EMI Instrumentation and System for derivation of the near/far-field distance.

5.68
Eq. (5.32) is in agreement with the definitions given in MIL-STD-449C. Thus, in order to insure that acceptable far-field conditions exist for EMI prediction, it is required that:

\[ R > \frac{\lambda^2}{\lambda} \text{ and } R > 3\lambda \]

Both conditions must be satisfied at all times. These transition-distance criteria are illustrated graphically in Fig. 5.29.

As one moves off main-beam axis, the near-field to far-field transition distance is reduced considerably. For high-gain antennas, the transition distance for this off-axis condition is determined from Fig. 5.30 in the following manner. First, locate the solid curve that corresponds to the aperture dimension, \( \lambda \), for the antenna of interest. Next, locate the intersection of this solid curve with the dashed curve that corresponds to the frequency of interest. The valid region of the solid curve lies above and to the left of the intersection with the frequency curve. For this region, the relationship between transition distance and angular displacement can be obtained directly from the solid curve. If the angular displacement is such that it lies in the invalid region of the solid curve, the near-field criteria given in Eq. (5.31) should be used to determine the near-field transition distance.
Use $R = 3 \lambda$. At 14 MHz, half wave dipole ($l < \lambda$) the far field is considered to be ~ 250 feet.
APPENDIX V
WAVE IMPEDANCE